

**Mathematical studies**  
**Standard level**  
**Paper 2**

Wednesday 13 May 2015 (afternoon)

1 hour 30 minutes

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

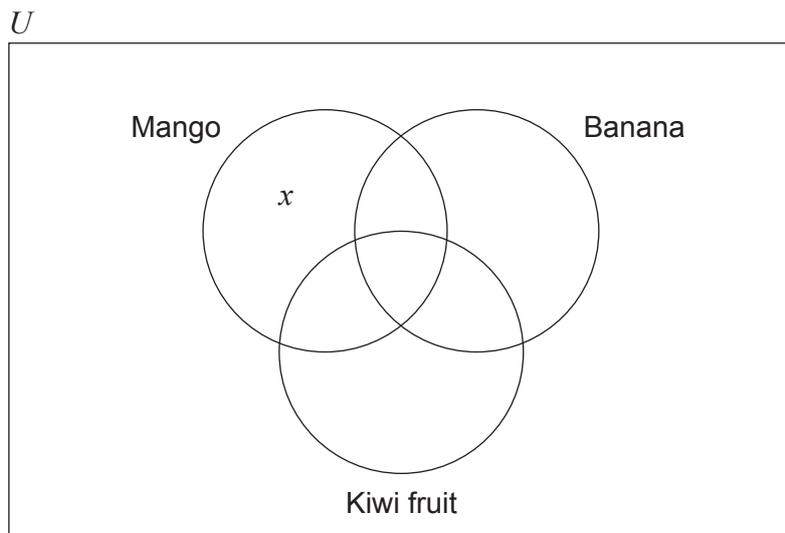
1. [Maximum mark: 16]

A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

- 15 like all three fruits
- 22 like mangoes and bananas
- 33 like mangoes and kiwi fruits
- 27 like bananas and kiwi fruits
- 8 like none of these three fruits
- $x$  like **only** mangoes

(a) **Copy** the following Venn diagram and correctly insert all values from the above information.

[3]



The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas.

(b) Complete your Venn diagram from part (a) with this additional information **in terms of  $x$** .

[2]

(c) Find the value of  $x$ .

[2]

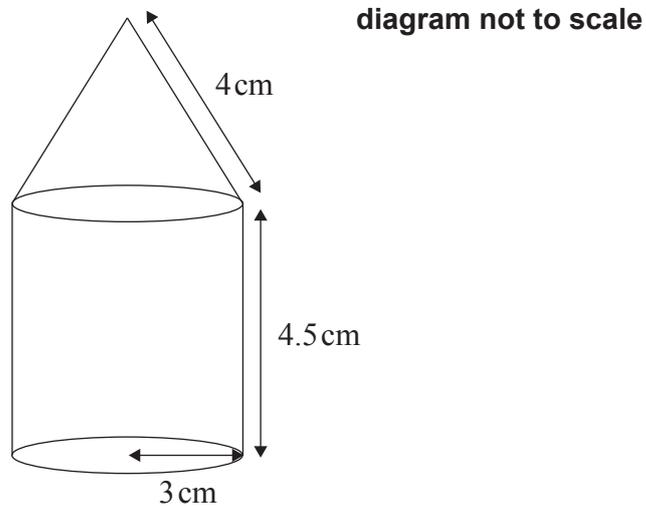
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**(Question 1 continued)**

- (d) Write down the number of customers who like
- (i) mangoes;
  - (ii) mangoes or bananas. [2]
- (e) A customer is chosen at random from the 100 customers. Find the probability that this customer
- (i) likes none of the three fruits;
  - (ii) likes only two of the fruits;
  - (iii) likes all three fruits given that the customer likes mangoes and bananas. [4]
- (f) Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits. [3]

2. [Maximum mark: 17]

The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

(a) (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures.

(ii) Calculate the volume of the perfume bottle.

[6]

The bottle contains  $125 \text{ cm}^3$  of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

(b) Find the height of the perfume in the bottle.

[2]

Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle.

(c) Find the **total** surface area of the perfume bottle.

[4]

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**(Question 2 continued)**

Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of  $7 \text{ cm}^2$ .

- (d) Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places.** [3]

Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is  $1 \text{ EUR} = 13.03 \text{ ZAR}$ .

- (e) Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places.** [2]

## 3. [Maximum mark: 12]

The sum of the first  $n$  terms of an arithmetic sequence is given by  $S_n = 6n + n^2$ .

(a) Write down the value of

(i)  $S_1$ ;

(ii)  $S_2$ .

[2]

The  $n^{\text{th}}$  term of the arithmetic sequence is given by  $u_n$ .

(b) Show that  $u_2 = 9$ .

[1]

(c) Find the common difference of the sequence.

[2]

(d) Find  $u_{10}$ .

[2]

(e) Find the lowest value of  $n$  for which  $u_n$  is greater than 1000.

[3]

(f) There is a value of  $n$  for which

$$u_1 + u_2 + \dots + u_n = 1512.$$

Find the value of  $n$ .

[2]

4. [Maximum mark: 12]

Daniel grows apples and chooses at random a sample of 100 apples from his harvest. He measures the diameters of the apples to the nearest cm. The following table shows the distribution of the diameters.

<b>Diameter (to the nearest cm)</b>	5	6	7	8	9
<b>Frequency</b>	15	27	33	17	8

- (a) Using your graphic display calculator, write down the value of
- (i) the mean of the diameters in this sample;
  - (ii) the standard deviation of the diameters in this sample. [3]

Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table.

<b>Classification</b>	<b>Diameter (cm)</b>
Small	Diameter < 6.5
Medium	$6.5 \leq \text{Diameter} < a$
Large	Diameter $\geq a$

- (b) Calculate the percentage of **small** apples in Daniel’s harvest. [3]

Of the apples harvested, 5% are **large** apples.

- (c) Find the value of  $a$ . [2]
- (d) Find the percentage of **medium** apples. [2]

This year, Daniel estimates that he will grow 100 000 apples.

- (e) Estimate the number of **large** apples that Daniel will grow this year. [2]

5. [Maximum mark: 17]

Consider the function  $f(x) = 0.5x^2 - \frac{8}{x}$ ,  $x \neq 0$ .

(a) Find  $f(-2)$ . [2]

(b) Find  $f'(x)$ . [3]

(c) Find the gradient of the graph of  $f$  at  $x = -2$ . [2]

Let  $T$  be the tangent to the graph of  $f$  at  $x = -2$ .

(d) Write down the equation of  $T$ . [2]

(e) Sketch the graph of  $f$  for  $-5 \leq x \leq 5$  and  $-20 \leq y \leq 20$ . [4]

(f) Draw  $T$  on your sketch. [2]

The tangent,  $T$ , intersects the graph of  $f$  at a second point, P.

(g) Use your graphic display calculator to find the coordinates of P. [2]

6. [Maximum mark: 16]

The following table shows the number of bicycles,  $x$ , produced daily by a factory and their total production cost,  $y$ , in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
<b>Number of bicycles, <math>x</math></b>	12	15	14	17	20	18	21
<b>Production cost, <math>y</math></b>	3900	4600	4100	5300	6000	5400	6000

- (a) (i) Write down the Pearson’s product–moment correlation coefficient,  $r$ , for these data.
- (ii) Hence comment on the result. [4]
- (b) Write down the equation of the regression line  $y$  on  $x$  for these data, in the form  $y = ax + b$ . [2]
- (c) Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. [3]
- All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**.
- (d) Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day. [2]
- (e) (i) Write down an expression for the total selling price of  $x$  bicycles.
- (ii) Write down an expression for the **profit** the factory makes when producing  $x$  bicycles on a particular day.
- (iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit. [5]
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